Actor-Based Models for Longitudinal Networks

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Actor-Based Models for Longitudinal Networks

Synonyms

Stochastic actor-based models, Agent-based models, Actor-oriented modelling

Glossary

Actors Nodes of the network graph.

Behaviour Changing characteristics of actors.

Covariates Variables which can depend on the actors (actor covariates) or on pairs of actors (dyadic covariates). They are considered "exogenous" variables in the sense that they are not determined by the stochastic process underlying the model.

Dyad Pair of actors of the network.

Dyadic indicator Binary variable indicating the presence or absence of a tie between two actors.

Effects Specifications of the objective function.

Longitudinal networks Repeated measures of networks over time.

Markov chain Stochastic process where the probability of future states given the present state does not depend on past states.

Method of Moments Statistical estimation method consisting of equating sample moments of a distribution with unobserved theoretic moments in order to get an approximation to the solutions of the likelihood equations.

Network Graph representing a relation on the set of actors with binary dyadic indicators which can be regarded as a state changing over time.

Network dynamics Study of longitudinal networks.

Objective function Function which determines probabilistically the dyadic changes made by the actors evaluating all information included in the connectivity structure of the actors.

Rate function Expected number of opportunities for change per unit of time.

Ties Relational connections between nodes of a network graph.

Introduction

The study of longitudinal networks has become a major topic of interest and dynamic modelling approaches have been pursued in much of social network analysis. Important applications range from friendship networks (see for example, Pearson and West (2003) and Burk et al (2007)) to inter-organizational networks (see for example, Brass et al (2004)). However many of the classical statistical models proposed have focused mainly on single static network analysis. One of the reasons why network dynamics was not tackled until a couple of decades ago is that the complex dependence structures that characterize networks could not allow an exact inferential calculations and estimation procedures cannot be dealt without computer simulation algorithms.

These powerful tools have allowed researchers to focus to the analysis of the underlying mechanisms that induce the characteristics of network dynamics from the "micro dynamics" such as the individual actor choices to the "macro properties" such as the network connectivity structure. Key research topics concern the structural positions of the actors, their connectivity evolution, belief development, friendship formation, diffusion of innovations, the spread of a particular behaviour, etc.

Modeling the dynamics of social networks is therefore of crucial importance but it is also extremely difficult, due to the temporal dependence, but also since network data, at any given time instance, are not composed of independent observations but each tie variable between two actors is dependent on the presence or absence of ties in the other dyads. Consequently the network dynamics is greatly affected by the global connectivity structure. For this reason standard statistical models cannot give an adequate representation of this dependence feature. Various models have been proposed for the statistical analysis of longitudinal social network data and some earlier reviews were given by Wasserman (1979) and Frank (1991).

An actor-oriented approach to this type of modelling was pioneered by Snijders Snijders (1996, 2001, 2005) and Snijders and van Duijn (1997) under the assumption of statistical dependence between observations evolving over time according to a continuous time Markov process. These models were originally designed to model the evolution of expressive networks consisting of individuals but they can provide a general framework for the analysis of many different kinds of relations. Some applications were presented by van de Bunt (1999), de Nooy (2002), Huisman and Steglich (2008) and van Duijn et al (2003). The actor-based models are a family of statistical models aiming to describe network dynamics according to some typical network dependencies such as reciprocation of ties, transitivity, etc. They represent one of the most prominent class of models for the analysis of network dynamics as they allow a flexible analysis of the complex social network dependencies among the actors over the time. In this context, the network dynamic is assumed to be driven by different effects modelled by network statistics which operate simultaneously. The stochastic process defined by these effects can provide a good representation of the changes of the network connectivity structure over time. The model parameter estimates allow one to understand the strength of the effects included in the model. The actor-based models are flexible as they allow to incorporate a wide variety of network statistics. The main objectives of this approach consist in representing a wide variety of effects or tendencies on network evolution, estimate parameters expressing such tendencies and test corresponding hypotheses so as to understand the structuring of social networks over time. These effects are various and can be created based on the application. The parameter estimates obtained from the inferential process can be used to simulate network structures compatible with the tendencies observed in the network changes under study. This chapter does not give a review of this literature concerning modelling approaches for longitudinal network data but it focuses only on the class of stochastic actor-based models. In this chapter we describe the basic features of the actor-based models by providing the theoretical assumptions and methodology requirements needed. We give some basic insights concerning the inferential analysis for the parameters and goodness of fit procedures. Next we carry out an illustration of the capabilities of these models through their application to an ethnographic study of community structure in a New England monastery by Sampson (1968). A brief discussion on the future directions is given at the end of the chapter.

Stochastic Actor-Based Models for Network Dynamics

Longitudinal social networks represent a useful tools for explaining the development over time of many different kinds of social relations (such as friendship, advice, communication) within a group of actors (such as people or organizations). These relations between actors or nodes are by nature subject to change at any time. The individual properties and behaviour of the nodes and the similarity characteristics of pairs of nodes can generally affect the topology structure of the network and therefore its evolution. For this reason the major difficulty in the analysis of social networks is that each connected dyad depends on the sub-graph connectivity structures of its nodes. Network dynamics can be driven by several different effects such as reciprocity, transitivity, homophily, etc. These tendencies or effects are responsible for the creation and termination of the ties between the actors of the network.

The stochastic actor-based models are a family of models designed to analyse the mechanisms which determines the change and the evolution of social networks by taking into account the strength of a wide variety of these effects described by some model parameters. The availability of statistical procedures for estimating the parameters and testing their significance allows us to understand the contribution and significativity of each single effect. Social networks can be represented by graphs and the ties represent the states of the relation between the nodes. Longitudinal networks are usually represented by panel data collected at different times.

Notation

The relational structure of a social network can be represented by a graph on a set of nodes $N = \{1, \ldots, n\}$ connected by dyadic variables represented as a $n \times n$ adjacency matrix X whose element $\{X_{ij}\}$ represents the relation between node i (sender) and j (receiver). The dyad $\{X_{ij}\}$ can take value 1 or 0 indicating the presence or absence of a tie going from node i to j. Self-ties and valued ties are not considered in this context, so by convention we set $x_{ii} = 0$. Here we use capital letters to denote random variables, and lower case letters to denote the corresponding observations.

Longitudinal dyads can be denoted by $X_{ij}(t)$ so as to indicate that the states are time-dependent and are collected in the random adjacency matrix $X(t)$. There can also be other variables that may influence the network and they are regarded as covariates. These can be depending on actors (e.g. the sex of actors) and are denoted by V_i, or dyads (e.g. spatial distance between two actors) and are denoted by W_{ij}.

Methodology

Stochastic actor-based models are typically concerned with directed networks whose states have the tendency to endure over time in order to satisfy the requirement of *gradual change*. This property is generally observed in many kinds of social relations such as friendship, trust, cooperation, etc. In this context, changes in the states of the network are generally assumed to be dependent on the current network states and not on the past ones. This means that all the relevant information for modelling the future state is assumed to be provided by the current state of the network. In statistical terms this means that the social network is a stochastic process with *Markov property*. This assumption is obviously an approximation which can be considered unrealistic in some cases.

Basic Assumptions

- The parameter t is continuous. In most applications, observations of the network are made at some discrete (generally small) number of outcomes of the process $X(t)$ which evolves in continuous time t. Actor-based models interpret this discrete time series of observed networks as the cumulative result of an observed sequence of elementary changes made by the actors between two consecutive observations. This continuous process is not observed and it is inferred from modelling. An analogous approach was proposed by Coleman (1964).

- $X(t)$ is a Markov process, that is, the conditional probability distribution of future states of the network depends on the past states only as a function of the present states. In other words, the states of the network at time t include all the knowledge for predicting future states of the network at time $t+1$. This assumption takes into account the tendency of ties to remain in place until some special event happens.
- At any given time t, no more than one dyadic variable $X_{ij}(t)$ can change. This assumption is related to the fact that changes of ties are mutually dependent only because tie changes will depend on the current global connectivity configuration of the network. The process is assumed to be decomposable into sequence of smallest possible changes.
- The process is actor-based in the sense that dyadic changes are made by the actors who create or drop a tie on the basis of their covariate attributes and their position in the network. The actor-based process is characterized by two stochastic sub-processes:
  - The *change opportunity process* models the frequency of the dyadic changes made by actors. The *rate function* is denoted by λ and represents the probability of the occurrence of a change in a given small time interval. This frequency rate can be constant or dependent on the nodal covariates or on the current network connectivity structure.
  - The *change determination process* models the choice of the *ego actor* who gets the opportunity to make a dyadic change. The ego actor may create or drop one outgoing tie or make no changes. The probability of the

choice is modelled by the *objective function* $f_i(x^0, x, v, w)$ which depends on the current state of the network x^0, the potential new state of the network x and the covariate attributes v (nodal) and w (dyadic).

Model Specifications and Estimation

Specification of the Objective Function

The objective function has a crucial role in actor-based models as it determines the rules of the actor behaviour in the network. When an opportunity for actor i occurs at a rate λ_i, the actor can change one of the outgoing dyadic variables X_{ij}, $\forall j \in N$ leading to a new state of the network x. The probability of this new state of the network is given by:

$$p_i(x|x^0, v, w) = \frac{\exp\{f_i(x^0, x, v, w)\}}{\sum_{x' \in C(x^0)} \exp\{f_i(x', x, v, w)\}}, \tag{1}$$

where $C(x^0)$ denotes the set of all possible networks resulting from a change in the current network x^0. The changes can be considered as determinations of new dyadic states of X_{ij} according to a multinomial logistic regression model. Similar to generalized linear models, the objective function is specified as a linear combination of a set of network statistics called *effects* $s_{ki}(x)$ (Figure 1) which correspond to "tendencies" driving the network dynamics:

$$f_i(\beta, x) = \sum_k \beta_k s_{ki}(x), \tag{2}$$

where β_k are statistical parameters indicating the impact of the corresponding network statistic on the dynamic process: the greater the value of the parameter the higher the probability of the corresponding network statistics to have an impact on the network dynamics and vice versa if the value of the parameter is negative. The choice of the set of effects to include in a model is generally theory-guided and it often depends on the application context. In the following section we will describe some of most used effects included in the objective function.

Basic Effects

Outdegree - Indegree These effects correspond to the number of outgoing and incoming ties for actor i respectively. These network statistics can be defined as:

$$s_{Oi}(x) = \sum_j x_{ij}, \qquad s_{Ii}(x) = \sum_j x_{ji}, \qquad i \neq j. \tag{3}$$

They give a measure of the position and popularity of actor i. If the corresponding parameters have a positive values the importance of these effects will tend to increase or stay high over time.

Reciprocity This effect corresponds to the number of reciprocated ties for actor i. The network statistic is defined as:

$$s_{Ri}(x) = \sum_j x_{ij} x_{ji}, \qquad i \neq j. \tag{4}$$

It gives a measure of the tendency toward reciprocation of choices referring to mutual tied: if i connects to j then j connects to i and vice versa. This effect shows significant positive evidence in many kind of networks such as friendship relational data.

Other degree-based effects may be formulated by taking into account more complicated features of the actor's behaviour. For example, actors may have preferences to connect to other actors based on both their own and other's degrees. In many context the degrees reflect status of an actor and therefore can play a crucial part in the tie formation and evolution.

Triadic structures These family of effects incorporate information involving three actors and they can measure two important features of the network such as *tran-*

sitivity and *transitivity closure*. Several network statistics can be formulated to analyse transitivity.

The **transitive ties** effect is defined as:

$$s_{TEi}(x) = \sum_h x_{ih} \max_j (x_{ij} x_{jh}), \qquad i \neq j \neq h, \tag{5}$$

and measures the number of configurations involving three actors in which $x_{ij} = 1$, $x_{jh} = 1$, and $x_{hi} = 1$. This effect models the tendency toward generalized exchange.

The **transitive triplets** effect is one of these and it is defined as:

$$s_{TTi}(x) = \sum_j x_{ij} \sum_h x_{ih} x_{hj}, \qquad i \neq j \neq h, \tag{6}$$

and measures transitivity for actor i by counting the number of actors j for which there is at least one intermediary h forming a transitive triplet.

The **three-cycles** effect is defined as:

$$s_{Ci}(x) = \sum_j x_{ij} \sum_h x_{jh} x_{hi}, \qquad i \neq j \neq h, \tag{7}$$

and measures the number of configurations involving three actors in which $x_{ij} = 1$, $x_{jh} = 1$, and $x_{hi} = 1$. This effect models the tendency toward generalized exchange.

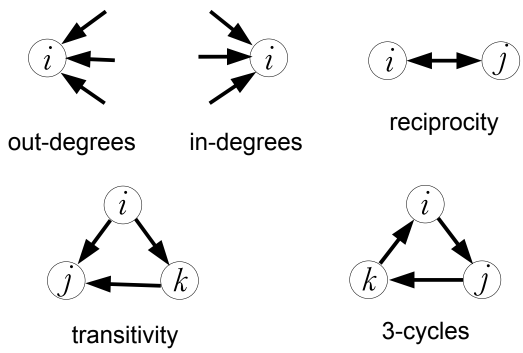

Fig. 1. Some of the most common network effects for stochastic actor-based models.

Covariate-based effects These are effects that take into account the information on the nodes (v_i) or ties (w_{ij}). For example, the **similarity** or **homophily** effect measures whether ties tend to occur more often between actors with similar values of V. The **ego** effect reflects the propensity of the actor to send ties, and leading to a correlation between v_i and out-degrees. The **alter** effect models the tendency of the popularity of the actor for receiving ties, and leading to a correlation between v_i and in-degrees. It is also possible to include dyadic attributes expressing different kind of features such as meeting opportunities or spatial propinquity, etc. More formulae of effects can be found in Snijders (2012).

Network and Behaviour Co-evolution

The network structure can have an impact on the behaviour and performance of the actors. For example actors can be influenced by their neighbours because of many different factors such as competition, cooperation, etc. and, in turn, influence other actors. This type of changing attributes are referred to as *behaviour*. The vector of attributes for actor i are no longer assumed to be constant over time and it is denoted by $z_i(t)$. The model for the network and behaviour co-evolution is defined as the stochastic process $(X(t), Z(t))$. Now the structure changes of $X(t)$ will depend on both the current network states $X(t)$ and behaviour states $Z(t)$ and, similarly, the behaviour changes of $Z(t)$ will depend on both the current network states $X(t)$ and behaviour states $Z(t)$. The behaviour process is characterized by a rate function λ^Z driving the frequency of changes and objective function f_i^Z which defines the probabilities of changes. The process $(X(t), Z(t))$ follow the assumptions of gradual change over time. At any given moment t no more than one of the set of variables $X_{ij}(t)$ and $Z_{ih}(t)$ can change. This means that there is no direct coordination between changes in ties and changes in behaviour and the dependence is made by the influence they have to each other. The objective function described above can be used to model the *behaviour* of the focal actor i or some of his neighbours and on his network position, etc. The so-called *shape, influence and position-dependent* effects allow to model the behaviour change. It is generally quite challenging to estimate a model which includes both dynamic and behaviour effects, for this reason in many cases it is advisable first to focus on fitting a good network dynamic model and subsequently include terms for the behaviour evolution process.

Data Requirements and Modeling Issues

In order to use stochastic actor-based models for longitudinal networks, the assumptions described above should be plausible from a practical point of view. For example, the total number of changes between two consecutive network observations has to be “large enough” so that the changes can actually provide the necessary information for estimating the parameters. At the same time, this amount of change does not have to be too high so as not to violate the assumption of gradually changing states. One of the implicit assumptions of the model is that the actors of the network have an homogeneous behaviour toward change so specific outlying behaviours are not taken into account.

Statistical Estimation

The distribution of the parameter estimates of the parameters β_k in the objective function are approximately normally distributed and therefore the significance of the parameters can be tested using the t-ratio testing context. Estimation procedures are possible by a variety of simulation-based methods. The Method of Moments proposed by Snijders (2001) operates by selecting a vector of statistics, one for each parameter coordinate to be estimated, and determining the parameter estimate as the parameter value for which the expected value of this vector of statistics equals the observed value. This estimation procedure is iterative and based on variants of the Robbins-Monro algorithm Robbins and Monro (1951). The convergence of this algorithm is generally quite good but it can be affected by the initial value. In many cases good starting points are obtained from estimates of simple models. Generally fitting complicated models may affect the convergence and the performance of the estimation algorithm and this may result in obtaining poor estimates. Moreover network statistics can be highly correlated by definition and this implies that also the parameter estimates can be strongly correlated. For these reasons forward selection procedures are preferred to backward ones. The estimation procedure is based on the principle that the first observed network is considered a starting point of the dynamic process so it is not modelled directly.

The parameters β_k of the objective function can be interpreted as weights of the model effects. It is important to check if the estimated model is able to describe the overall features of the stochastic process. Goodness of fit for stochastic network models is generally based on the comparison of a set of data simulated from the estimated model with the observed data (Hunter et al, 2008). For each network statistic the percentile at which the observed value is located in the distribution of simulated network statistics is used as a test. Obviously it is recommended to consider also effects that are not directly included in the model estimated so as to have an overall picture of the fit of model to the data.

Example: Dynamics of Social Relations in a Monastery

The Sampson monastery network dataset (Sampson, 1968) consists of social relations among a set of 18 monk-novitiates preparing to enter a monastery in New England. There are three separate adjacency matrices representing liking relations at three points in time (Figure 2). Based on observations and analyses, the monks can be partitioned into four groups: Young Turks, Loyal Opposition, Outcasts, and an interstitial group. The Loyal Opposition consists of the novices who entered the monastery first. The Young Turks arrived later, in a period of change. They questioned practices in the monastery, which the members of the Loyal Opposition defended. Some novices did not take sides in this debate, so they are labelled “interstitial”. The Outcasts are novices who were not accepted in the group.

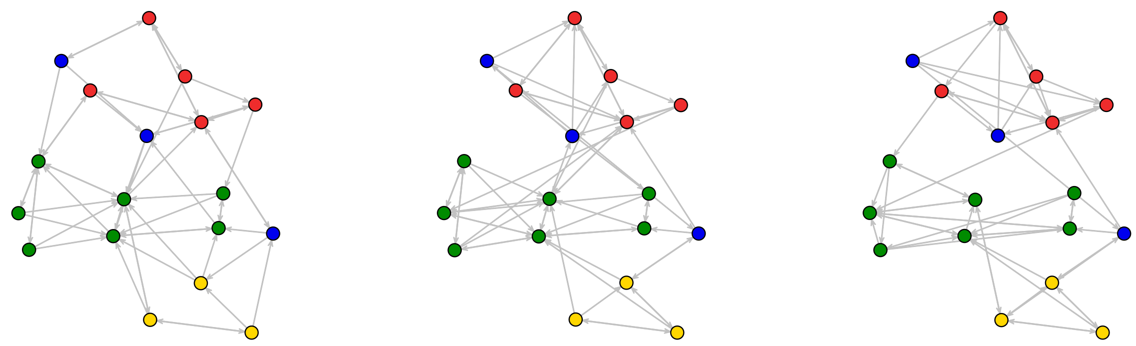

Fig. 2. Sampson Monastery network graph at three time points. Colours represent the actors membership covariates: green actors belong to Young Turks, red actors belong to the Loyal Opposition, yellow actors belong to Outcasts, and blue actors belong to interstitial group.

In this explanatory example we propose two models. In the first model we decided to include only structural network effects: `outdegree` measuring the network graph connection density, `reciprocity` measuring the importance of mutual ties, `transitive triplets` measuring transitivity, and `3-cycles` measuring the tendency toward clustering. The second model includes the previous network effects plus two nodal covariate-based terms which take into account the group membership of the actors: covariate-related tendency for mutual ties (`group similarity x reciprocity`) defined by the number of preferences for mutual ties with actors that have similar values on a certain individual level nodal covariate and a covariate-related identity (`same group`) defined by the number of ties of actor i to all other actors j's who have exactly the same value on the covariate, in this case, group membership. All the calculation were done using the RSiena package version 1.1-212 (Snijders, 2012) and the parameter estimates and standard errors of both models are reported in Table 1.

Effect	Estimate	S.E.
	Model 1	
Network rate function		
`Rate parameter` (period 1)	3.5692	0.7472
`Rate parameter` (period 2)	2.6102	0.4637
Network objective function		
`outdegree`	−1.4080	0.2131
`reciprocity`	1.2313	0.2880
`transitive triplets`	0.2898	0.1350
`3-cycles`	−0.1475	0.2213
	Model 2	
Network rate function		
`Rate parameter` (period 1)	3.7309	0.7143
`Rate parameter` (period 2)	2.8616	0.5490
Network objective function		
`outdegree`	−1.5110	0.2023
`reciprocity`	0.8109	0.3030
`transitive triplets`	0.1486	0.1389
`3-cycles`	−0.3796	0.2294
`group similarity x reciprocity`	0.4154	0.7772
`same group`	1.2887	0.3253

Table 1. Parameter estimates with standard errors of the actor-based models used for modelling the Sampson monastery longitudinal networks.

The analysis of the first model confirms that there is an overall low level of outdegrees and a high level of reciprocity, as indicated by the two significant `outdegree` and `reciprocity` parameter estimates; there is no evidence of transitive closure as indicated by the estimates of `transitive triplets`. In the second model, the covariate based effect concerning the `group similarity x reciprocity` is significant meaning that reciprocity is high between actors belonging to the same group. Rate parameters indicate that the liking relations between monks have a peak in the first period between the first two observations and then decrease slightly in the second period. These differences are obviously reflecting the amounts of change observed between two consecutive network observations. It can be concluded that monks tended to like monks belonging to the same group and this tendency tends to reinforce itself over time determining the loss of across-group ties and the creation of within-group ties.

In this simple example we have considered that dropping a tie is the opposite of creating a new one. However in many contexts this is not plausible. In order to take this into account it is possible to consider another component of the objective function: the *endowment* function. This operates only for the termination of ties and not for their creation.

Tie-based Approaches

The modelling approach presented in this chapter is actor-based in the sense that the dynamic process and the statistical inference is driven by the behaviour of each single actor of the network. However a tie-based version of the longitudinal model was proposed by Snijders (2006) and corresponds to exponential random graph models (ERGMs, see Robins et al (2007) for a review). These models are best suited for analysing the global topological properties of static networks who are assumed to be in equilibrium. This approach makes it difficult to infer the development of longitudinal networks when the structural trend represented by tie-based network statistics is not discernible. In other words, the tie-based network statistics used in the exponential random graph modelling context are inadequate to micro structural changes which are not affecting the global topology of the graph. For this reason, the actor-based models represent a richer class of models which do not require the sequence of observed networks to be in equilibrium and model the global features of the evolving network by taking into account its micro changes over time. Moreover the computational cost of fitting exponential random graph models can be substantial, particularly for large networks.

Modeling Multiple Networks

The actor-oriented models can also be applied for longitudinal multiple network observations. Suppose that at each time point M different dependent relational structures are observed on the same set of N actors. An example is friendship and trust relations for the same set of individuals. The actor-based models now is defined as:

$$Y(t) = (Y_1(t), \ldots, Y_M(t)) \tag{8}$$

Rate and objective functions can be defined separately for each dependent network. Some effects expressing dependencies between multiple networks based on composition of relations are: *direct dependence* measuring the propensity that actors who are connected by a certain relation tend to be connected by another relation; *cross-network dependence* measuring the propensity that some relational ties tend to be reciprocated by ties of a different kind; etc.

Future Directions

The area of statistical modelling of network dynamics is of growing in development. This chapter has given an introduction to the flexible family of stochastic actor-based models for analysing longitudinal networks. These models can be used to formulate and test hypotheses concerning the evolution of networks in order to obtain a useful representation of the dynamic behaviour of the network structure by measuring the strength of various effects and estimating the corresponding parameters.

The advantage of this modelling approach is that parameter estimates identify a model which provides an easy interpretation of the effects driving the dynamic change in the network structure that are clear reflections of the patterns and regularities that can be derived from the analysis of static social networks. For the purposes of statistical inference, actor-based models provide an important tool for representing the dependencies between the ties of the network and the behaviour of the actors over time.

Several applications have demonstrated the usefulness of these models and have provided a better understanding of the interpretability of the results. In particular actor-based models have been proven to be useful in the context of questions about selection and influence in social relational data. Various modelling extensions have recently been proposed for example by Checkley and Steglich (2007) and van de Bunt and Groenewegen (2007). From a inferential point of view other estimation procured have been recently proposed: maximum likelihood estimation by Snijders et al (2010) and Bayesian estimation by Koskinen and Snijders (2007). The software SIENA ("Simulation Investigation for Empirical Network Analysis") (Ripley and Snijders, 2011) and its R version RSiena (Snijders, 2012) for the statistical analysis of network data provides a very useful tool for practitioners and applied scientists.

Cross-references

Evolution of social networks, exponential random graph models, modeling and analysis of spatio-temporal social networks, models of social networks, R packages for social network analysis, Siena, dynamics and evolution models for social networks.

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